Part I: SHORT ANSWER

You only write your final answer. The maximum points for each number are 5 points.

1. Let $a, b \geq 0$ and $a * b = \frac{a+b+1}{ab+12}$

Find the value of $0 * \left(1 * \left(2 * \left(\ldots\left(2020 * (2021 * 2022)\right)\ldots\right)\right)\right)$.

Answer_____

2. John wants to paint every positive integer with blue or red such that if two different numbers have the same colour, then the sum of those two numbers also has the same colour. In how many ways can John do this?

Answer_____

3. Suppose that x, y are integers. Find all possible values of $x + y$,

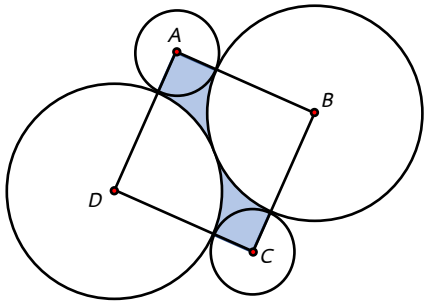
where $9 - 2xy - 3x - 6y = 0$.

Answer_____

4. Given two circles tangent internally at P and a chord AB of the larger circle tangent the smaller circle at Q, determine the ratio between $\angle APB$ and $\angle BPQ$.

Answer_____

5. Let four circles has center at A, B, C, and D are tangent to other circles as shown below. Circles A and C have the same radius. Circles B and D have the same radius. The angles of A, B, C, and D are right angles. If the area of shaded region is 65 unit^2, find the area of quadrilateral ABCD. (Answer in term of π)



Answer_____

Part II: ESSAY

Write your answers completely with step–by–step. The maximum points for each number are 15 points.

6. Determine the largest integer which is less than $\frac{3^{2022}+2^{2022}}{3^{2018}+2^{2018}}$.

7. Balls are numbered 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Three groups of balls can be selected so that the sum of each group is 11. For example, one arrangement is {1, 10}, {0, 2, 3, 6}, {4, 7}. Including the example, how many arrangements are possible?

8. Solve the equation

$$1+\frac{1}{x}+\frac{(x+1)(y+1)}{xy}+\frac{(x+1)(y+1)(z+1)}{xyz}=0$$

.

9. Given that a, b, c are positive real numbers such that $abc = 32$, find the minimum possible value of $4a^2 + 4ac + 2b^2 + c^2$.

10. Let $PQRS$ be a parallelogram of center O. Points M and N are the midpoints of OQ and RS, respectively. Prove that if triangles PQR and PMN are similar, then $PQRS$ is a square.

ANSWER

No.	ANSWER
1	$\frac{23}{233}$
2	6
3	-23, -12, -11, 2, 3, 14
4	2 : 1
5	$\frac{260}{4-\pi}$
6	80
7	60
8	x = −1 and y, z can be any real number except y=0, z=0 y = $\frac{-(2z+1)}{3z+1}$ and x, z can be any real number except x=0, z=0, z=$-\frac{1}{3}$, z = $-\frac{1}{2}$
9	96
10	