1. For a 4×4 matrix A, if $\det A = 6$, then find $\det(AdjA)$.

2. Find the product of all roots of the equation $x^{\log_5 x} - 625x^3 = 0$?

3. Given $x = -1+\sqrt{2}i$ and a matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1-x^2 & x^2 \\ 1 & -1+x & x^2 \end{bmatrix}$.

   Find $-2\left(\det A\right)^2$.

4. If $\frac{\sin^4 x}{4} + \frac{\cos^4 x}{5} = \frac{1}{9}$, then find the value of $\tan^2 x$.

5. Assume $2\sin\theta\cos\theta + \sin\theta - \sqrt{3}\cos\theta = \frac{\sqrt{3}}{2}$ for $0 \le \theta \le 2\pi$,

   find $\sin^2\theta + \cos\theta$.

6. In an arithmetic sequence $u_1 = \log_b\left(\frac{x}{y}\right)$ and $u_2 = 9\log_b y$, where $b > 1$

   and $x = b^{100}$ and $y = b^{98}$, respectively. Find the value of $\sum_{n=1}^{50} \frac{u_n}{10000}$.

7. Evaluate $\sum_{n=1}^{10}(-1)^{n+1}\frac{n^2+n+1}{n!}+\sum_{n=1}^{2}\frac{5n+1}{(5n)!}$.

8. Evaluate the sum

$$\sum_{k=0}^{3}\frac{120}{(3-k)!(3+k)!}.$$

9. Let α,β and γ be complex solutions of the equation $z^3+z^2-7z+65=0$, where $i=\sqrt{-1}$. Find the value of $4(|\alpha|^2+|\beta|^2+|\gamma|^2)$.

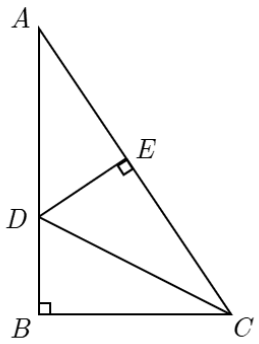
10. Evaluate $\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\ldots+\frac{2023}{2021!+2022!+2023!}+\frac{1}{2023!}$.

11. Let x,y,z be complex numbers such that $x+y+z=3, x^2+y^2+z^2=5$, and $xyz=2$. Evaluate $\left|\frac{1}{xy+2z-2}+\frac{1}{yz+2x-2}+\frac{1}{xz+2y-2}\right|$.

12. Consider the polynomial function $f(x)=x^4+ax^3+bx^2+cx+d$, where a,b,c,d are real numbers. Two distinct roots of $f(x)=0$ are given by $k-k^2i$ and k^2-ki, where k is a real number and $i=\sqrt{-1}$. The remainder when $f(x)$ is divided by x is 12100. Find k^2.

13. Consider the function $f(x)=\left(3x^3+4x^2-5x+14\right)\left(3x^2+mx+n\right)$, where m and n are real numbers. It is given that the sum of all roots of the equation $f(x)=0$ is $-\frac{51}{9}$, and that the product of all roots is $\frac{14}{3}$. Find $m+n$.

14. Let ΔABC be a right triangle whose angle B is the right angle. Points D and E lie on the side AB and AC, respectively. Suppose that a line DC divides the angle BCA into 2 angles which are $B\hat{C}D=30^\circ$ and $E\hat{C}D=45^\circ$, a line DE is perpendicular to the side AC, and $DC=4$. If $AB=a+\sqrt{b}$, then find $a+b$.



15. Let $ABCD$ be a trapezoid which has vertices $A(1,4), B(7,4), C(3,-2)$, and $D(-1,1)$. If the height of this trapezoid is equal to $\frac{a\sqrt{b}}{b}$, where $\gcd(a,b)=1$, then find the value of ab.

16. How many possible three-digit codes are there to unlock iPhone by using the number 0-9, if the first digit cannot begin with one or three, the last two digits cannot be the same number, and the last digit cannot be one?

17. A and B are events such that $P(A) = 0.65, P(B) = 0.36,$ and $P(A \cup B)' = 0.24$ Find $P(A' \cup B')$.

18. Find the number of ways to put 8 donuts in 10 distinct boxes, if the donuts are indistinguishable, and no box can have more than one donut?

19. Suppose that a candy shop has four different kinds of candy. How many different ways can you pick eight candies?

20. The bag contains 12 orange marbles and 8 white marbles. A marble is drawn from the bag and its color is recorded, and then a fair coin is flipped. If the coin lands on tails, then a second marble is drawn from the bag without replacing the first marble. If the coin lands on heads, then the first marble is replaced in the bag before a second marble is drawn. Find the probability that the second marble is orange.

21. Given an arithmetic series

$\log_5 5^{400} + \log_5 5^{380} + \log_5 5^{360} + \ldots + \log_5 5^{x} = 4180.$

Solve for x, where $x > 0$.

22. James is a secret agent working at the coordinate $\begin{pmatrix} 7 \\ 13 \end{pmatrix}$. He would like to send his coordinate to the headquarters. However, he must encode his coordinates before sending by using an encoding matrix $\begin{pmatrix} 1 & -1 \\ 5 & -2 \end{pmatrix}$ for securing his coordinate from enemies. Encoding is the product of the encoding matrix and the real coordinate to get another coordinate. If he sends an encoded coordinate $\begin{pmatrix} x \\ y \end{pmatrix}$ to the headquarters, then find the value of $x^2 + y^2$.

23. Let $ABCD$ be a parallelogram which has vertices $A(2,3), B(-1,2)$, and $C(1,-1)$. Find the area of this parallelogram.

24. If the number is selected at random from the set of all four-digit numbers in which the sum of digits is equal to 34, what is the probability that the number will be divisible by 11?

25. How many integer solutions are there to the equation
$x_1 + x_2 + x_3 + x_4 = 7$, where $x_1 \geq 1, x_2 \geq 1, x_3 \geq 1, x_4 \geq 2$?

ANSWERS M.4-6

No.	Answer	No.	Answer
1	216.00	14	54.00
2	125.00	15	234.00
3	144.00	16	648.00
4	0.80	17	0.75
5	0.25	18	45.00
6	107.81	19	165.00
7	1.05	20	0.60
8	7.00	21	40.00
9	204.00	22	117.00
10	0.50	23	11.00
11	1.50	24	0.40
12	10.00	25	10.00
13	10.00		