1. Prof. Yosu has three dark bags. Bag no 1 contains three red balls and one yellow ball. Bag no 2 contains one red ball and three yellow balls. Bag no 3 contains two red balls and two yellow balls. Aci randomly selects one of the bags, and randomly chooses one ball to see the color of the ball.
If the first ball had been red and she continues to randomly selects a second ball from the same bag as the first ball. What is the probability that the second ball is red?

Answer

2. Katherine Johnson wrote a sequence of positive integers. In her sequence, the sum of every three consecutive numbers is an even number and the sum of every four numbers is an odd number. How many numbers at most had Katherine Johnson written in her sequence?

Answer

3. There are ten girls playing basketball. Among them are Ada Lovelace, Emmy Noether, Florence Nightingale, Radia Perlman.
They will split into two teams each consists of 5 girls with the following conditions
   - Florence wants to play in the same team as Emmy
   - Ada does not want to play in the same team as Radia

How many different ways are there to split these 10 girls into 2 teams?

Answer

4. Let $ABCD$ be a square. Points E and F are located on sides AB and BC respectively. Given that $EF = AE + CF$
Find the measure of $\angle EDF$

Answer

5. In parallelogram $ABCD$, diagonals AC and BD intersect at E. Let M and N be the midpoints of BE and CD respectively. Given that triangles ABC and AMN are similar, show that $ABCD$ is a square

Answer

Answer Key

1.	$\frac{73}{144}$
2.	5
3.	30
4.	45°
5.	As $\Delta ABC \sim \Delta AMN$, $\frac{AB}{AC} = \frac{AM}{AN}$ and $\angle MAN = \angle BAC$. But we also have $\angle BAM = \angle CAN$. So, it implies $\Delta BAM \sim \Delta CAN$. Then $\frac{BA}{BM} = \frac{CA}{CN}$ and $\angle ABM = \angle ACN$. So, $ABCD$ is a cyclic quadrilateral. Then we may conclude that $ABCD$ is a rectangle. Next, note that $BM = \frac{1}{4}BD = \frac{1}{4}AC$ and $CN = \frac{1}{2}AB$. Therefore $\frac{BA}{BM} = \frac{CA}{CN} \Leftrightarrow \frac{BA}{\frac{1}{4}AC} = \frac{CA}{\frac{1}{2}AB} \Leftrightarrow 2AB^2 = AC^2 = AB^2 + BC^2$. Hence $AB^2 = BC^2$. So, $ABCD$ is a square.