Part I: SHORT ANSWER

You only write your final answer. The maximum points for each number are 5 points.

1. In a sequence of 10 terms, the first term is 1, the second term is x, and each term after the second is the sum of the previous two terms. For example, if $x = 27$, the sequence would be 1, 27, 28, 55, 83, 138, 221, 359, 580, 939. For some values of x, the number 2022 appears in the sequence. If x is a positive integer, what is the sum of all the values of x for which 2022 appears in the sequence?

Answer_____

2. Sara has a set of unit cubes. She glues together 4 of these cubes to make a solid. Cube faces must coincide when they are glued together. Each of the 4 cubes must have a face that coincides with a face of at least one of the other 3 cubes. The number of unique solids that Sara can make using 4 cubes is .....

Answer_____

3. Mike constructs all non-congruent triangles with perimeter 57 and integer side lengths x, y, z, such that $x < y < z$. How many such triangles are there?

Answer_____

4. Anna has five different numbers. She wants to assign these numbers to the variables p, q, r, s, and t so that $s < t$, $p < s$, $r < t$, and $q < s$. In how many ways can she assign these numbers?

Answer_____

5. Five players (A, B, C, D, E) take part in a Blokus tournament. Every two players must play (as partners) against every other two players. Any two given players can be partners not more than once per day. What is the least number of days needed for this tournament?

Answer_____

Part II: ESSAY

Write your answers completely with step–by–step. The maximum points for each number are 15 points.

6. Let $ABCD$ be a rectangle and E, F be points on the segments BC and DC, respectively, such that $\angle DAF = \angle FAE$. Show that if $DF + BE = AE$, then $ABCD$ is a square.

7. A circle is divided into 11 sectors. 99 coins are located in these sectors, with 9 coins in each sector. At every move, you can move a coin from a sector to one of two neighbour sectors. (Two sectors are called **neighbours** if they are adjoined along a segment.) Is it possible to move all coins into one sector in exactly 2565 moves?

8. There is a pond which has 35 apple trees on its right-hand side standing in a row. The distance of the first tree from the pond is 15 *m* and each tree is 7 *m* apart. Elena wants to water each apple tree, one tree at a time, and return to the pond to get water for the next one. Starting from the pond, What is the total distance that Elena will cover in order to water all the apple trees?

9. Rita puts five boxes in a row, each containing at least one ball. Two balls are said to be 'neighbours' if they are either in the same box or they are in adjacent boxes. Each ball has exactly five or exactly ten neighbours. How many balls are there in all five boxes?

10. A pastry chef makes a cake by using the following ingredients:

Ingredient	Quantity (gram)
Flour	50
Sugar	65
Vegetable oil	30
Milk	30
Egg	125
Butter	275

Macronutrient	Caloric equivalent per gram
Protein	4
Carbohydrates	4
Fat	9

A chef makes a cylindrical cake with diameter 7 inches and height 1.5 inches. Then he separates the cake into 8 pieces to sell at his bakery. The customer, who is on a diet, wants to buy some cake. He can eat the dessert that has at most 0.5 kilocalories. How many at most the number of pieces of cake that the customer can eat?

ANSWER

No.	ANSWER
1	4,043
2	8
3	61
4	8
5	4
6	
7	Yes, it is possible.
8	9,127
9	17
10	A piece of cake